

# NEURAL NETWORK BASED MODELS FOR SURFACE ROUGHNESS OBTAINED DURING ELECTROCHEMICAL MACHINING OF SG IRON.

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## ABSTRACT

Only a few parameters that affect the machining process in ECM are controllable. It is clearly established from reported works that results reported in literature cannot be extrapolated. So for any new material - electrolyte combination and machining conditions experiments need to be conducted to predict the effects of process parameters on machined geometry. For effective exploitation of ECM for machining SG Iron it is essential to develop models for predicting the nature of surface that will be generated. Two widely used methods for correlating controllable process parameters and surface roughness parameters are Design of Experiments and Artificial Neural Networks (ANN). The back-propagation algorithm is used widely as a learning algorithm in feed-forward multilayer neural network. A large data set is usually necessary to train the ANNs. However, generating a large number of data for training ANN is not only time consuming but very expensive. Hence to reduce the cost of developing NN models it is decided to train the network with only those thirteen data that are used for developing Box Behnken design based models. The three configurations of the ANN considered are 3-10-10-1, 3-20-20-20-1 and 3-39-39-39-1. Because of the small data set the deviations noted between predictions obtained using Box-Behnken models and ANNs are high. Denoising is done in two stages using Multiscale Principal Component Analysis (PCA) and dropping the high frequency coefficients by filtering with Daubechies wavelet. The first stage denoising carried out using PCA has resulted in significant reduction in deviation (error). After second level denoising the errors have reduces further. Of the three network configurations studied lowest configuration gave the best results in majority of the cases and where as for the highest configuration, the denoising effect is small. Not only the error is reduced but the number of points with large errors has reduced substantially. The differences between the outputs from the neural network model and Box Behnken models are well within the confidence intervals calculated from ANOVA.

**KEYWORDS:** Artificial Neural Network, Box Behnken Design, Denoising, Electrochemical Machining, Principal Component Analysis, SG Iron, Sa, Sq, Sz, Ssk, Sku, Smmr, Smvr, SHtp, Wavelet Analysis

## **INTRODUCTION**

Electrochemical machining (ECM) can be used to machine complex features in hard and difficult to machine electrically conductive materials with negligible tool wear, reasonable accuracy and acceptable surface finish. Only a few parameters that affect the machining process are controllable. ECM results of only a few combinations of electrolyte and work-piece material, under specific machining conditions have been reported. It is clearly established that results reported in literature cannot be extrapolated. So for any new material - electrolyte combination and machining conditions experiments need to be conducted to predict the effects of process parameters on machined geometry.

Little information is available on machining of SG Iron by electrochemical machining process [1]. For commercial exploitation of ECM for machining SG Iron it is essential to develop models for predicting the nature of surface that will be generated.

Models based on experimental results may be generated using many methods. Two widely used methods are Design of Experiments and Artificial Neural Networks (ANN).

The back-propagation algorithm is used widely as a learning algorithm in feed-forward multilayer neural network. The back-propagation is applied to feed-forward ANNs with one or more than one hidden layers. Based on this algorithm, the network learns a distributed associative map between the input and output layers. The network is trained by repeatedly using a set of input-target data. The network learns the relationship between the input and target data by adjusting the weights to minimize the error between the input and target values. The major advantages of BPNN over the regression technique are: [2, 3]

- No mathematical model is required
- Capable of modeling highly non linear relationship
- Capable of using dispersed data in the solution domain
- Existing models can be refined using new data sets easily through training
- In spite of the advantages it has over the regression models, there are a number of drawbacks. Some of them are:
- No theoretical basis exists to determine the number of hidden layers and number of neurons therein. Different configurations of BPNN have great effects on the predicted results.
- Scaling of the data to suit the non-linearity function (usually sigmoid) has great effect on predicted results.

Developing models based on design of experiments are not easy. Many times data need to be transformed to make the variable normal. Finding out the transformation equations are not straight forward. Secondly, selecting the design of experiments is a challenging task. Thirdly, the models cannot be upgraded even if addition information is received at a later date. To overcome some of these problems back-propagation networks are constructed and trained using the Neural Network Toolbox of the software package MATLAB [4]. For effective training of the NN a large number of data is usually necessary. For example it is reported [5] that 442 data were used to train the NN and 110 data are used for testing the network. Experiments are very costly and time consuming. Hence to reduce the cost of developing NN models it is decided to train the network with only those thirteen data that are used for developing Box Behnken design based models.

Development of the NN models:

Box-Behnken design [6] is used for developing the second order mathematical models. The general model is given below.

$$Y = B_0 + B_1T + B_2V + B_3G + B_{11}T^2 + B_{22}V^2 + B_{33}G^2 + B_{12}TV + B_{13}TG + B_{23}VG$$

Where, T - machining time, V - applied potential, G - gap between tool and work-piece

For simplifying the recording of the conditions of the experiments and processing of the experimental data, the upper, lower and intermediate levels of the variables are coded as +1, -1& 0, respectively. The experimental levels and the

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design matrix are shown in **Table 1, 2**. Two electrolyte solutions used are KCl solution (250 grams of KCl / litre of tap water) and NaNO<sub>3</sub> solution (400 grams/litre of tap water).

| Variables                | Symbol | Low    | Level | Intermedi | ate Level | High Level |       |
|--------------------------|--------|--------|-------|-----------|-----------|------------|-------|
|                          | v      | Actual | Coded | Actual    | Coded     | Actual     | Coded |
| TIME (minutes)           | Т      | 2      | -1.0  | 3         | 0         | 4          | +1.0  |
| POTENTIAL (volt)         | V      | 15     | -1.0  | 20        | 0         | 25         | +1.0  |
| INTER ELECTRODE GAP (mm) | G      | 0.64   | -1.0  | 0.96      | 0         | 1.28       | +1.0  |

Table 1: The Actual and Coded Values of Different Variables

Specification of work-piece material is given in **Table 3**. The details of experimentation, the models and the statistical analyses are given in ref [7]. Besides checking statistical validity of the models, experimental verification is also carried out [7].

| CL M.   | Variables |    |    |  |  |  |  |
|---------|-----------|----|----|--|--|--|--|
| 51. NO. | Т         | V  | G  |  |  |  |  |
| 1       | -1        | -1 | 0  |  |  |  |  |
| 2       | +1        | -1 | 0  |  |  |  |  |
| 3       | -1        | +1 | 0  |  |  |  |  |
| 4       | +1        | +1 | 0  |  |  |  |  |
| 5       | -1        | 0  | -1 |  |  |  |  |
| 6       | +1        | 0  | -1 |  |  |  |  |
| 7       | -1        | 0  | +1 |  |  |  |  |
| 8       | +1        | 0  | +1 |  |  |  |  |
| 9       | 0         | -1 | -1 |  |  |  |  |
| 10      | 0         | +1 | -1 |  |  |  |  |
| 11      | 0         | -1 | +1 |  |  |  |  |
| 12      | 0         | +1 | +1 |  |  |  |  |
| 13      | 0         | 0  | 0  |  |  |  |  |
| 14      | 0         | 0  | 0  |  |  |  |  |
| 15      | 0         | 0  | 0  |  |  |  |  |

**Table 2: Design Matrix** 

Table 3: Work-Piece Material Specification (SG Iron)

|       | Chemic | DIIN  | Nadularity 0/ |        |      |               |  |  |
|-------|--------|-------|---------------|--------|------|---------------|--|--|
| %C    | %Si    | %Mn   | %S            | %P     | DIIN | Nouliarity 70 |  |  |
| 3.60- | 2.30-  | 0.35- | 0.014-        | 0.083- | 170  | 58 24         |  |  |
| 3.63  | 2.38   | 0.36  | 0.013         | 0.080  | 1/9  | 38.24         |  |  |

The confidence intervals for surface roughness values calculated from the models are also calculated. The variance for the mean predicted values can be calculated using equation 1 [8].

$$V(\widehat{y}) = \frac{\sigma^2}{3} - \frac{5}{24}\sigma^2(\sum x_{io}^2) + \frac{13}{48}\sigma^2(\sum x_{io}^4) + \frac{7}{24}\sigma^2(\sum x_{io}^2 x_{jo}^2)$$
(1)

The confidence interval is calculated based on the equation (2).

$$y_{io} = \hat{y}_{io} \pm \sqrt{S_{io}^2 \times F_{0.05,1,5}}$$
 where,  $S_{io}^2 = S_t^2 [1 + V(\hat{y})]$  (2)

Where,  $(\hat{y})$ : variance of estimated response at a point given by  $(x_{1o}, x_{20}, x_{30})$ ,  $\sigma$ : mean square error,  $S_t^2$  = Mean square of residual,  $\hat{y}_{io}$  = calculated response at a point given by  $(x_{1o}, x_{20}, x_{30})$ . F : F-ratio.

The coefficients of the second order models for surface roughness parameters viz. Sq (Root-Mean-Square (RMS) Deviation of the Surface,  $\mu$ m), Sku (Kurtosis of the Topography Height Distribution), Ssk (Skewness of the Topography Height Distribution), SHtp (Surface section height difference(20% - 80%) and the statistical analyses are given in **Table 4**. The experimental conditions and results obtained from the models along with confidence intervals are given in **Table 5**. The experimental values are within the confidence interval hence the models can be used for optimization.

|      |                        |          | KCl Ele  | ectrolyte |          | NaNO <sub>3</sub> Electrolyte |          |          |          |  |
|------|------------------------|----------|----------|-----------|----------|-------------------------------|----------|----------|----------|--|
|      |                        | Sq       | Ssk      | Šku       | SHtp     | Sq                            | Ssk      | Sku      | SHtp     |  |
| _    | Bo                     | 8.30667  | -0.18087 | 2.84333   | 13.43334 | 6.78000                       | -0.32540 | 3.16000  | 12.6000  |  |
| S    | B <sub>1</sub>         | 0.44000  | 0.06925  | 0.18750   | 2.14625  | 0.38875                       | -0.40650 | -0.44750 | 0.32250  |  |
| effi | <b>B</b> <sub>2</sub>  | 1.12625  | -0.22128 | -0.2387   | 2.11500  | 0.12750                       | -0.40037 | 1.95250  | -0.64250 |  |
| cie  | <b>B</b> <sub>3</sub>  | 0.08125  | -0.19798 | 0.22875   | 0.44375  | -1.62125                      | -0.07462 | 0.49000  | -3.39000 |  |
| nts  | B <sub>11</sub>        | 0.01666  | -0.28504 | 0.19083   | -0.30792 | -0.42000                      | 0.70158  | -0.34625 | -0.48375 |  |
| Q    | B <sub>22</sub>        | 2.06916  | 0.15891  | 0.31333   | 4.57458  | -3.50250                      | -0.45017 | 1.68375  | -7.93375 |  |
| T    | B <sub>33</sub>        | -3.08084 | -0.00009 | -0.3367   | -4.83792 | 1.15500                       | -0.31517 | -0.03625 | 2.74125  |  |
| ıe l | <b>B</b> <sub>12</sub> | -1.25750 | 0.36600  | 0.68250   | -1.60000 | 0.69250                       | -0.51400 | -0.92750 | 0.58250  |  |
| Μo   | B <sub>13</sub>        | 0.55250  | -0.29150 | 0.12750   | 0.00250  | 3.02500                       | -0.21200 | -0.52250 | 6.59250  |  |
| del  | B <sub>23</sub>        | 1.29000  | 0.09155  | 0.30000   | 2.03000  | 2.03750                       | -0.31125 | 0.30750  | 3.39750  |  |
| s D  | F <sub>RATIO</sub>     | 0.82413  | 0.05233  | 0.50843   | 0.35424  | 9.44613                       | 0.27768  | 0.33677  | 0.23678  |  |
| eve  | $\sigma^2$             | 0.27373  | 0.08892  | 0.02243   | 2.94333  | 0.05230                       | 0.10201  | 0.27181  | 3.87000  |  |
| ole  | $\mathbb{R}^2$         | 98.5071  | 91.5377  | 98.2455   | 96.8652  | 98.7993                       | 96.26523 | 95.3927  | 98.2579  |  |
| ped  | $R^{2}_{(adj)}$        | 95.8199  | 76.3056  | 95.0874   | 91.2227  | 96.6380                       | 89.54265 | 87.0998  | 95.1223  |  |
| _    | $R^{2}_{(pred)}$       | 85.2914  | 72.4920  | 85.6117   | 77.9881  | 81.8758                       | 76.49660 | 68.3718  | 89.8026  |  |

 

 Table 4: The Coefficients of the Models Developed and the Statistical Model Parameters for KCl & Nano3 Electrolyte

| Table 5: Model | Validation | (KCl | <b>Electrolyte</b> ) |
|----------------|------------|------|----------------------|
|----------------|------------|------|----------------------|

| ECM Parameters         |        |             | ]    | From Exp   | n Experiment |                 |                 | From Model       |        |                 |                 |                  |
|------------------------|--------|-------------|------|------------|--------------|-----------------|-----------------|------------------|--------|-----------------|-----------------|------------------|
| Sl. No                 |        | Т           | V    | G          | Sq           | S <sub>sk</sub> | S <sub>ku</sub> | S <sub>HTp</sub> | Sq     | S <sub>sk</sub> | S <sub>ku</sub> | S <sub>HTp</sub> |
|                        | coded  | 0.15        | -0.4 | -1.0       |              |                 |                 |                  |        |                 |                 |                  |
| 1                      | octual | 3.15        | 18   | 0.64 mm    | 4.99         | 0.262           | 2.51            | 9.08             | 5.6    | 0.199           | 2.516           | 9.26             |
|                        | actual | min.        | v    | 0.04 11111 |              |                 |                 |                  |        |                 |                 |                  |
| Confidnce interval (±) |        |             |      |            |              |                 | 1.515           | 0.6              | 0.385  | 4.11            |                 |                  |
|                        | coded  | 0.5         | -0.2 | 1.0        |              |                 |                 |                  |        |                 |                 |                  |
| 2                      | octual | 3.5         | 19   | 1.28       | 4.99         | -0.55           | 3.06            | 8.58             | 5.533  | -0.5656         | 2.873           | 9.55             |
|                        | actual | min.        | v    | mm         |              |                 |                 |                  |        |                 |                 |                  |
| Confidnce interval (±) |        |             |      |            |              |                 |                 | 1.5268           | 0.6043 | 0.388           | 4.14            |                  |
|                        | coded  | -1.0        | 0.5  | 0.34375    | 8.52         | -0.259          | 2.7             | 14.8             | 9.288  | -0.7412         | 2.511           | 13.909           |
| 3                      | actual | 3.5<br>min. | 19 v | 1.28 mm    |              |                 |                 |                  |        |                 |                 |                  |
| Confidnce interval (±) |        |             |      |            |              |                 | 1.535           | 0.6075           | 0.390  | 4.165           |                 |                  |

Using the experimental data the NN models are trained. The thirteen experimental data for each surface roughness parameters are used to train the NN models. The configurations of neural networks trained are given in **Table 6**.

| <b>BPNN</b> Abbreviations | Network Configuration |
|---------------------------|-----------------------|
| BPNN - N1                 | 3 - 10 - 10 - 10 - 1  |
| BPNN - N2                 | 3 - 20 - 20 - 20 - 1  |
| BPNN – N3                 | 3 - 39 - 39 - 39 - 1  |

Table 6: Configuration of the Neural Networks Trained

Nonlinear functions chosen are tan-sigmoid and purelin. Training algorithm used is Levenberg-Marquardt. Stopping criteria used is 300 epochs or error tolerance of 0.0001. For every network configuration thirty new starts are

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considered. The network is trained with random weights and average output is calculated. The average output is used for analyses. For validation of the neural network models developed, the models are tested with 8669 input conditions in coded form within the design space of sphere of radius of  $\sqrt{2}$ . The outputs from NN and outputs from the Box Behnken models for the same input conditions are compared.

Analysis of ANN Results: Figure 1 shows the absolute deviation (error) between ANN models and Statistical model. As the statistical models are experimentally validated hence, the outputs are taken to be standard. The deviations (errors) noted are quite high. Similar trends are seen also in other cases (Figure 4, 7, 9, 11, 13, 15& 17). As a very few data (13 nos.) is used for fitting the neural network models containing a large number of neurons hence the chance of noise is high. To remove the noise a two stage strategy is used. In the first stage Principal Component Analysis is used to denoise and in the second stage wavelet decomposition is used to denoise the models.



Figure 1: Absolute Errors w.r.t. Statistical Model (Sq\_KCl)



Figure 2: Absolute Errors w.r.t. Statistical Model\_1<sup>st</sup> Stage De-Noising (Sq\_KCl)



Figure 3: Absolute Errors w.r.t. Statistical Model\_2<sup>nd</sup> Stage De-Noising (Sq\_KCl)



Figure 4: Absolute Errors w.r.t. Statistical Model (Sq\_NaNO<sub>3</sub>



Figure 5: Absolute Errors w.r.t. Statistical Model\_1<sup>st</sup> Stage De-Noising (Sq\_NaNO<sub>3</sub>)



Figure 6: Absolute Errors w.r.t. Statistical Model\_2<sup>nd</sup> Stage De-Noising (Sq\_NaNO<sub>3</sub>)



Figure 7: Absolute Errors w.r.t. Statistical Model (Sku\_KCl)



Figure 8: Absolute Errors w.r.t. Statistical Model\_2<sup>st</sup> Stage De-Noising (Sku\_KCl)



Figure 9: Absolute Errors w.r.t. Statistical Model (Sku\_NaNO<sub>3</sub>)



Figure 10: Absolute Errors w.r.t. Statistical Model\_2<sup>nd</sup> Stage De-Noising (Sku\_NaNO<sub>3</sub>)



Figure 11: Absolute Errors w.r.t. Statistical Model (Ssk\_KCl)



Figure 12: Absolute Errors w.r.t. Statistical Model\_2<sup>nd</sup> Stage De-Noising (Ssk\_KCl)



Figure 13: Absolute Errors w.r.t. Statistical Model (Ssk\_NaNO<sub>3</sub>)



Figure 14: Absolute Errors w.r.t. Statistical Model\_2<sup>nd</sup> Stage De-Noising (Ssk\_NaNO<sub>3</sub>)



Figure 15: Absolute Errors w.r.t. Statistical Model (SHtp\_KCl)



Figure 16: Absolute Errors w.r.t. Statistical Model\_2<sup>nd</sup> Stage De-Noising (SHtp\_KCl)



Figure 17: Absolute Errors w.r.t. Statistical Model (SHtp\_NaNO<sub>3</sub>)



Figure 18: Absolute Errors w.r.t. Statistical Model\_2<sup>nd</sup> Stage De-Noising (SHtp\_NaNO<sub>3</sub>)

## **Denoising the ANN Model**

Denoising is done in two stages using : Multiscale Principal Component Analysis and dropping the high frequency coefficients by filtering with Daubechies wavelet (DB6)[9]. Multivariate denoising using wavelets is used to denoise the model obtained from neural network [10-11]. The aim of multiscale PCA is to reconstruct, starting from a multivariate signal and using a simple representation at each resolution level, a simplified multivariate signal. The multiscale principal component generalizes the normal PCA of a multivariate signal represented as a matrix by performing a PCA on the matrices of details of different levels simultaneously. A PCA is also performed on the coarser approximation coefficients matrix in the wavelet domain as well as on the final reconstructed matrix [11-12]. By selecting the numbers of retained principal components, interesting simplified signals can be reconstructed [13]. A simplified multivariate signal is obtained starting from a multivariate signal obtained from NN and using a simple representation at each resolution level. Kaiser's [14] rule is used to keeps the components associated with eigenvalues greater the mean of all eigenvalues. The Kaiser rule is to drop all components with eigenvalues under 1.0, this being the eigenvalue equal to the information accounted for by an average single item. A variation of this method has been created where confidence intervals are calculated for each eigenvalue and only factors which have the entire confidence interval greater than 1.0 are retained [15-16].

## Algorithm:

- Read a multivariate signal.
- Set the wavelet parameters such as level of decomposition and mother wavelet.
- Select the number of retained principal components using Kaiser's rule.
- Perform a simple multiscale PCA.
- Improve the obtained result by retaining less principal components.
- Suppress the details at levels 1 to n, update the numbers of retained principal components selected by Kaiser's rule.
- Perform multiscale PCA again.
- Estimate Performance parameters
- Display the original Signal, Noisy Signal and Denoised signals.

The Dropping of the high frequency coefficients [9] is used for decomposition of response. Sixth order decomposition is carried out. Sixth order low pass coefficients are used for reconstruction of response and the high pass coefficients are filtered out.

## **RESULTS AND DISCUSSIONS**

The two stage denoising has reduced the error substantially. The first stage denoising carried out using PCA has resulted in significant reduction in deviation (error). It can be observed from **Figure 2 & 5** that not only the error is reduced but the number of points with large errors have reduced substantially. This can be interpreted from the density variation between the original and these figures. After second level denoising the errors have reduces further (**Figure 3, 6, 8, 10, 12, 14, 16 & 18**). The differences between the outputs from the neural network model and Box Behnken models are well within the confidence intervals calculated from ANOVA.

It may be concluded that useful BP-ANN models can be developed based on a small number of data points placed on the boundary and centre of the design space after two stages of denoising the models. Of the three network configurations studied lowest configuration N1 gave the best results in majority of the cases and where as for the highest configuration N3 the denoising effect is small.

# CONCLUSIONS

- It is possible to develop effective NN models using a small set of data. The data points may be selected based of statistical design of experiments.
- Two stage denoising has effectively reduced the noise in NN models.
- Confidence interval for the predicted response may be calculated based on a few experiments at the center of the design space. NN models along with confidence interval will be a very powerful tool.

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